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# EVALUATION OF A SIX-COMPONENT THRUST STAND

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R. K. STROME, LT, USAF

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## TECHNICAL REPORT AFRPL-TR-69-151

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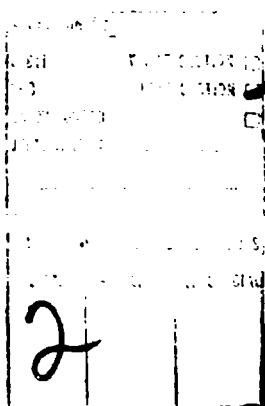
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EVALUATION OF A SIX-COMPONENT THRUST STAND

R. K. Strome, Lt, USAF

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## FOREWORD

This report was prepared by the Motor Component Development Branch, Solid Rocket Division, Air Force Rocket Propulsion Laboratory (AFRPL). The calibration and verification of the thrust stand was conducted at the AFRPL under Project 305903AMG, Solid Rocket Hardware Design and Evaluation (SRHDE), during the period November 1968 to January 1969, with Lt John R. Ellison as Project Engineer. The calibration was monitored by Lt Richard K. Strome.

This report has been reviewed and approved.

CHARLES R. COOKE  
Chief, Solid Rocket Division  
Air Force Rocket Propulsion Laboratory

## ABSTRACT

This report describes the calibration and an error analysis of the 6-component 50,000-pound Gilmore Rocket Motor Thrust Stand located on Pad 2, Air Force Rocket Propulsion Laboratory, Solid Test Area (1-32). The purpose of this report is to establish stand accuracy and reproducibility for a calibrated static test loading.

Errors considered primarily resulted from thrust stand misalignment, propellant mass flow of the motor, and cross-axis interactions. Dynamic effects resulting from vibration were not included in the detailed analysis but are briefly discussed.

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## SECTION I

### INTRODUCTION

A six-component thrust stand was designed by Gilmore Industries for use with an 84-inch-diameter test motor. The 84-inch-diameter burn surface and typical uncured propellant formulations produce thrust levels up to 50,000 pounds, and typical maximum burn durations of 60 seconds. The weight of a fully loaded motor is approximately 50,000 pounds. With the thrust stand framework, the maximum initial weight is approximately 60,000 pounds.

The Gilmore thrust stand helps fulfill thrust vector control (TVC) system testing requirements. The importance of accurately determining the thrust vector produced by a TVC system led to the error analysis that was performed on the Gilmore thrust stand. The error analysis serves to estimate the expected accuracy of the measured thrust and side force vectors produced by a TVC system.

Conventional assembly and calibration techniques were employed on the stand. In addition, a simulated firing loading condition (subsequently referred to as the "verification" test) of the stand was created to evaluate the accuracy of the stand. A study of this data was conducted to determine an estimate of uncertainty to assign to measurements made with this thrust stand. This report contains the methods, assumptions, and results of that study.

## SECTION II

### THRUST STAND EVALUATION

#### A. THRUST STAND DESCRIPTION

The Gilmore stand for the 84-inch-diameter Char motor is a vertical six-component thrust stand capable of measuring 50,000 pounds of thrust and side forces up to 10,000 pounds. The stand has an axial load cell and five side force load cells. Three 10,000-pound load cells are connected to the aft mounting ring. The remaining two 10,000-pound load cells are connected to the bottom of the thrust spider next to the axial load cell. The load cell designated with the subscript C (Figure 1) are part of the calibration system and are only employed during calibration.

With the geometry illustrated in Figure 1, a Y side force of 5,000 pounds applied at the top will theoretically produce a force of 4796 pounds in the upper load cells and 204 pounds in the lower load cells. (This example illustrates the inadequacy of load cell error estimates based on total measurement range). The four calibration cells are used one at a time to apply known loads near the individual load cells and develop a correlation between cell output and applied loads as shown in Figure 2. In the verification test, load distributions similar to firing conditions are achieved by introducing loads into the thrust stand motor assembly with a secondary standard load cell installed into the fixture as shown in Figure 3. The verification effort permits the correlation of load readings of the individual load cells with induced loads in all the side load cells (axial force was applied separately).

The ideal relationship between the thrust forces produced during the rocket firing and the individual load cell readings can be determined from a forces-and-moments balance analysis. Appendix I contains this

analysis and the results are repeated here to illustrate the operation of the test stand. The geometry of the load cells are shown in Figure 1. Using the nomenclature of Figure 1 the side forces  $F_x$  and  $F_y$  are given by:

$$F_x = -F_{x1} - F_{x3} - F_{x2}$$

$$F_y = F_{y1} + F_{y2}$$

The Z axis thrust magnitude  $F_z$  is given directly by the Z load cell. The total thrust force vector ( $T_v$ ) is:  $T_v = (F_z^2 + F_x^2 + F_y^2)^{1/2}$ .

The thrust vector  $T_v$  is, in general, at an angle  $\phi$  from the motor center-line and an angle  $\theta$  from the X axis in the X-Y plane, measured clockwise. These angles are defined by:

$$\phi = \text{arc tan } F_z / T_v$$

$$\theta = \text{arc tan } (F_y / F_x)$$

## B. THRUST STAND CALIBRATION

The thrust stand calibration was accomplished according to procedures outlined in the Gilmore manual. The thrust stand was calibrated to correspond with a specific set of test conditions which were a side thrust force less than 3,000 pounds and a maximum axial thrust of 30,000 pounds. These values were derived from a five-degree thrust vector orientation and an axial thrust of about 25,000 pounds.

The calibration was performed twice, once prior to propellant loading and verification testing and once after the firing. The first calibration was performed after a partial trial run to adjust the ranges of the digital acquisition system. For maximum sensitivity, thrust stand adjustments were made before the calibration to insure accurate results. The adjustments were very minor, being a simple tightening of all threaded connectors in the side force load trains. The first calibration was done to

verify proper functioning and provide input for an error analysis. The second calibration was performed to verify the repeatability of the first calibration. The calibration equipment (Figure 2) consisted of a highly accurate load cell and electrical readout box installed in series with a two-way hydraulic ram. A single-calibration ram assembly was installed in sequence at locations  $Y_{1c}$ ,  $Y_{2c}$ ,  $X_{1c}$ ,  $X_{2c}$ , and  $Z_c$ . The maximum loads for each position are shown in Table I.

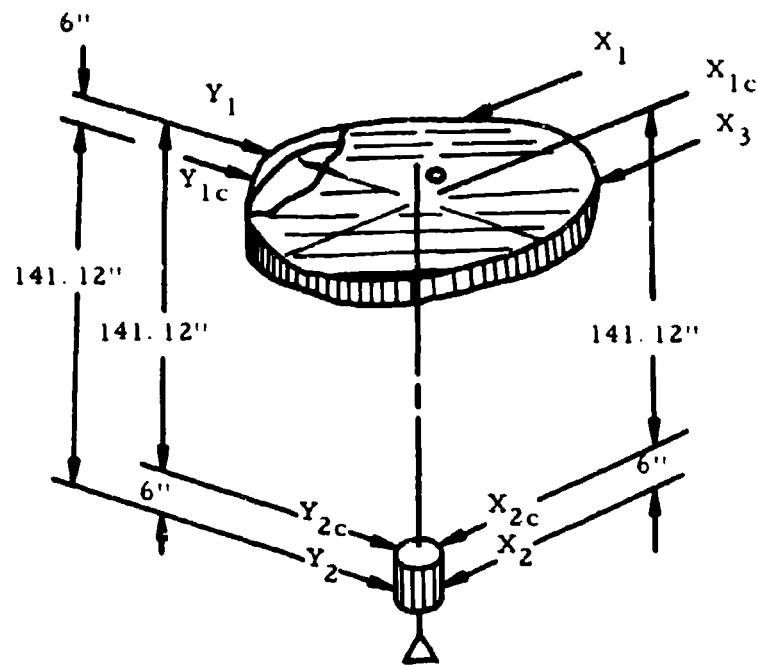
TABLE I. CALIBRATION LOADS

STATION	MAXIMUM CALIBRATION LOAD (POUNDS)
$X_{1c}$	5000
$X_{2c}$	5000
$Y_{1c}$	5000
$Y_{2c}$	5000
$Z_c$	40,000

Side thrust calibration loads were applied in 20% steps from zero to maximum tension value, through zero to an equal compression value and then returned to zero. At each step (registered on readout box and controlled by the hand-pump hydraulic power supply) the force measured by each load cell was simultaneously recorded by the digital data acquisition system. The "up and down" run was repeated three times at each calibration location before progression to the next.

Data obtained from the first calibration attempt led to the following observations:

1. Z axis - linear and reproducible, no hysteresis



X PLANE CALIBRATION				Y PLANE CALIBRATION		
Calib. Force	$X_1$	$X_2$	$X_3$	Calib. Force	$Y_1$	$Y_2$
$X_{1c}$	$-0.5 X_{1c}$	0	$-0.5 X_{1c}$	$Y_{1c}$	$-0.9592 Y_{1c}$	$-0.0408 X_{1c}$
$X_{2c}$	$-0.0204 X_{2c}$	$-0.9592 X_{2c}$	$-0.0204 X_{2c}$	$Y_{2c}$	$-0.0408 Y_{2c}$	$-0.9592 Y_{2c}$

Figure 1. Thrust Stand Dimensions and Theoretical Output Force Distribution

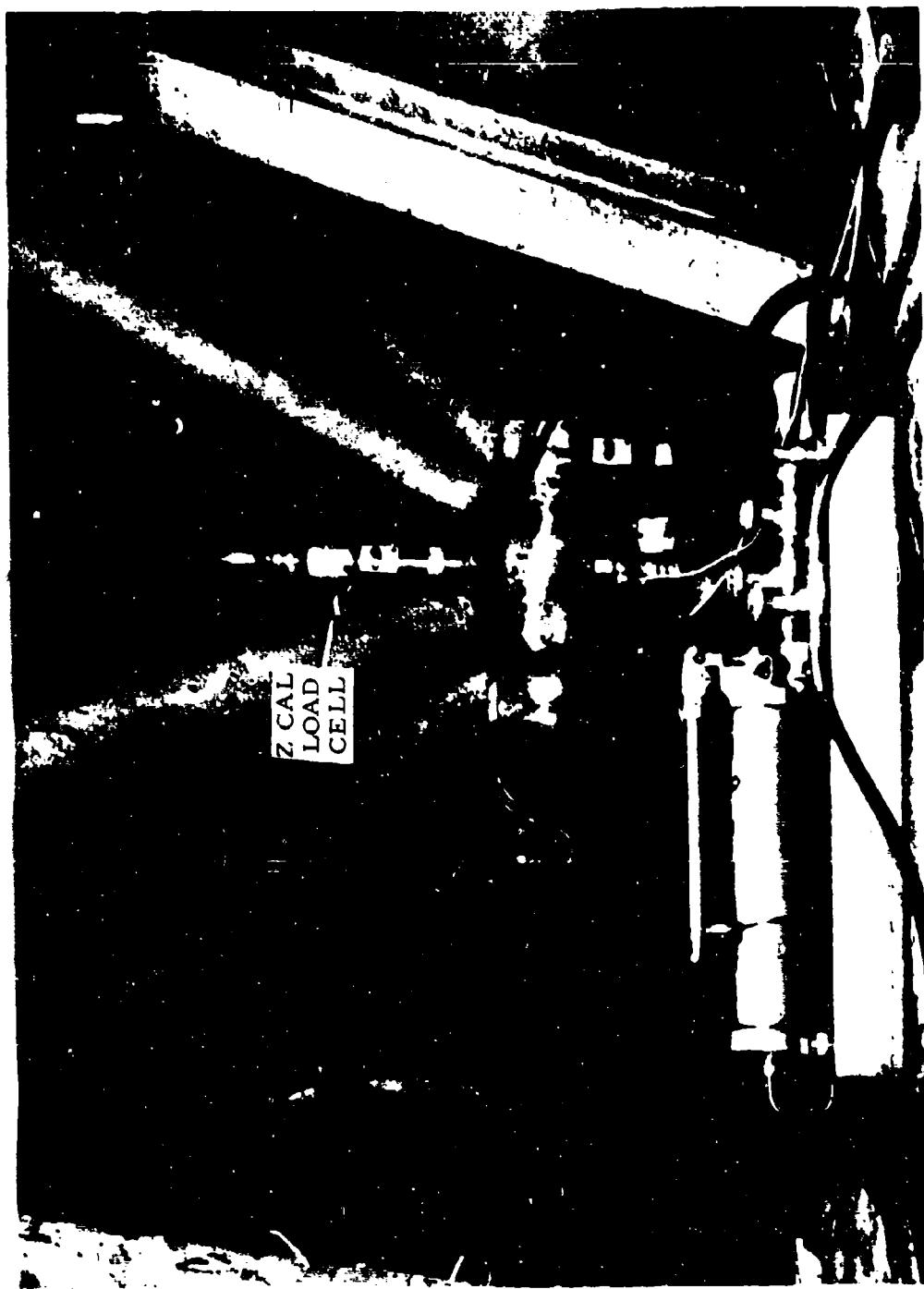


Figure 2. Z Calibration Load Cell



Figure 3. Thrust Stand and Verification Installation

2. X axis and Y axis - (a) each run linear, (b) no hysteresis, (c) third runs reproducible when compared with second runs, and (d) some zero-shift encountered when going from tension to compression as typified by Figure 4.

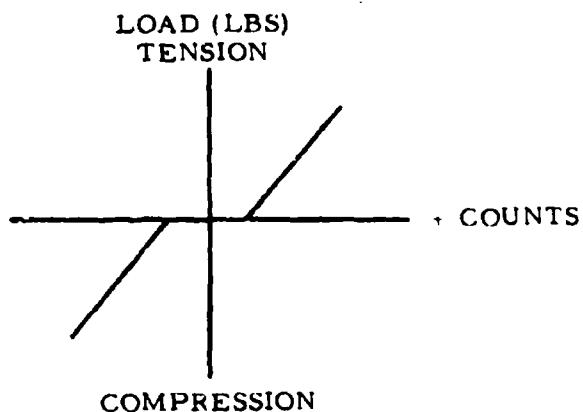


Figure 4. Calibration Counts versus Load

Data obtained from the second calibration lead to the following observations:

1. Z axis - linear, no hysteresis, reproducible and similar to the first calibration.

2. Y axis - linear, no hysteresis, reproducible and similar to the first calibration except the  $Y_2$  compression run. This run was nonlinear from 0 to 2000 pounds. The linearity was constant from 2000 to 5000 lbs. The  $Y_2$  axis contributed little to the measurement of side force because the side force was generated at the top of the motor near the nozzle. From the table on Figure 1, it can be seen that if a force was applied at  $Y_{1c}$ , less than 4.2% of the total load was measured by the  $Y_2$  load cell. During a firing, the  $Y_2$  load cell would measure less than 20% of the total load because of its position. The nonlinearity was less than 5% from the first calibration curve from 0 to 1500 lbs and therefore, the nonlinearity of  $Y_2$  contributed about 1% to the total error. Figure 5 shows where  $Y_2$  was nonlinear.

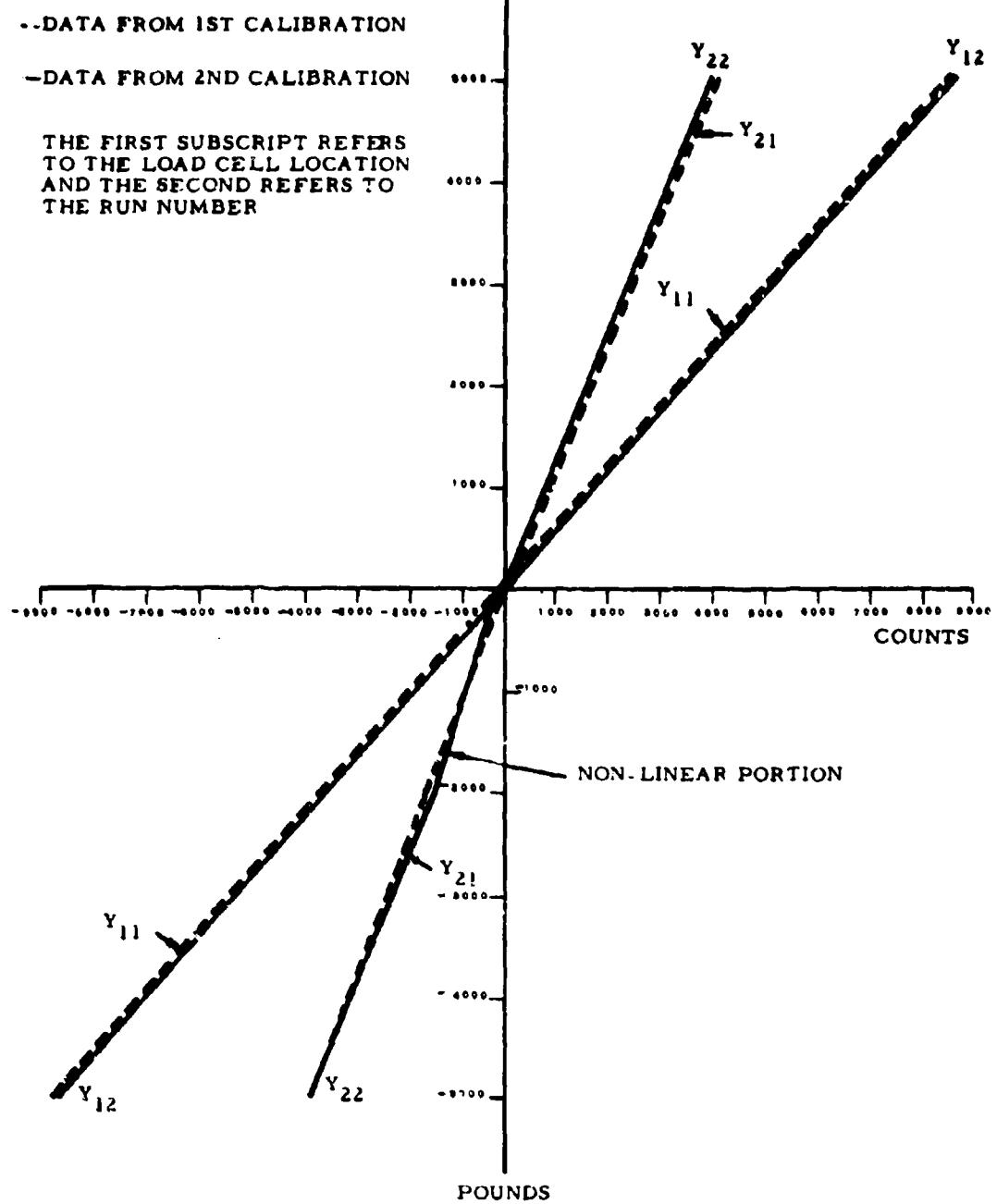


Figure 5. Comparison of First and Second Calibration, Y Load Cell

3. X axis - each run linear, no hysteresis, and reproducible similar to the first calibration except the  $X_2$  axis in compression was nonlinear from 3000 to 5000 pounds. The maximum nonlinearity was about 10% at 5000 pounds. The same situation exists in  $X_2$  as in  $Y_2$  except there were two load cells on top instead of one as in the Y direction.  $X_2$  measures less than 15% of the total side force, therefore the error contributed to the nonlinearity of  $X_2$  was less than 1.5% at 5000 lbs side force. Since the maximum anticipated side force was less than 3000 lbs, the nonlinearity will not affect side force measurement in this range. Figure 6 shows the nonlinearity of  $X_2$ .

The probable reason that the  $X_2$  and  $Y_2$  steps in compression were nonlinear at certain loads was because the load cells were not bolted to the structure as tight as possible. About 5 months elapsed between the 2 calibrations. During this time, the motor was subjected to the following: a fire from a nearby motor malfunction, significant rain and wind, as well as unknown disturbances by normal test crew operations around the motor.

#### C. THRUST STAND VERIFICATION

The purpose of the verification test was to determine how accurately the thrust stand functioned as a measurement system. Each of the load cells had been calibrated individually in the laboratory and their individual accuracies determined. The effect of the thrust stand geometry on mis-alignments of load cells and indications of force was unknown, so the system accuracy was determined with the verification test.

Instead of conducting a firing, which normally follows a stand calibration, a precisely applied load simulated the thrust and side force (in the three directions). The verification loads were applied near the predicted thrust vector action point on the Z axis. This was accomplished by the use

of a highly accurate (secondary standard) load cell and hydraulic ram positioned on top of the motor (Figure 3). Since a vertical force of 40,000 lbs and a horizontal force of 5000 lbs was to be applied, there had to be a structure to transmit the load to the motor. A verification structure (Figure 3) was fabricated to accomplish this. In this manner known forces were applied in the X, Y and Z directions.

For the evaluation of the X, Y and Z axes, simultaneous readings of all load cells were recorded, reduced with the calibration equations and compared to the known values of loads applied. The following standard loads were applied to the stand in 20% steps:

X direction	5000 lbs.
Y direction	5000 lbs.
Z direction	40000 lbs.

Since the verification structure was not completely rigid, it deflected about 0.25 inches when a vertical force of 40,000 lbs was applied. When the horizontal force of 5,000 lbs was applied the structure deflected in bending. Due to this, it was difficult to apply standard loads exactly in the desired plane. If the load was misaligned by a given angle, the error in indicated applied force would vary as the cosine of the angle. It would take an angle of 8 degrees to produce a 1% error. The structure was very difficult to analyze because it moved in a complex manner. Thus, the amount of error that was attributed to the movement of the stand was difficult to determine exactly. The amount of misalignment was believed to be small (less than 1/2 degree) because the surfaces were accurately machined and initially aligned with accurate machinist's levels. The evaluation of the verification results confirmed that the thrust stand produced acceptable results with an error of less than 2%.

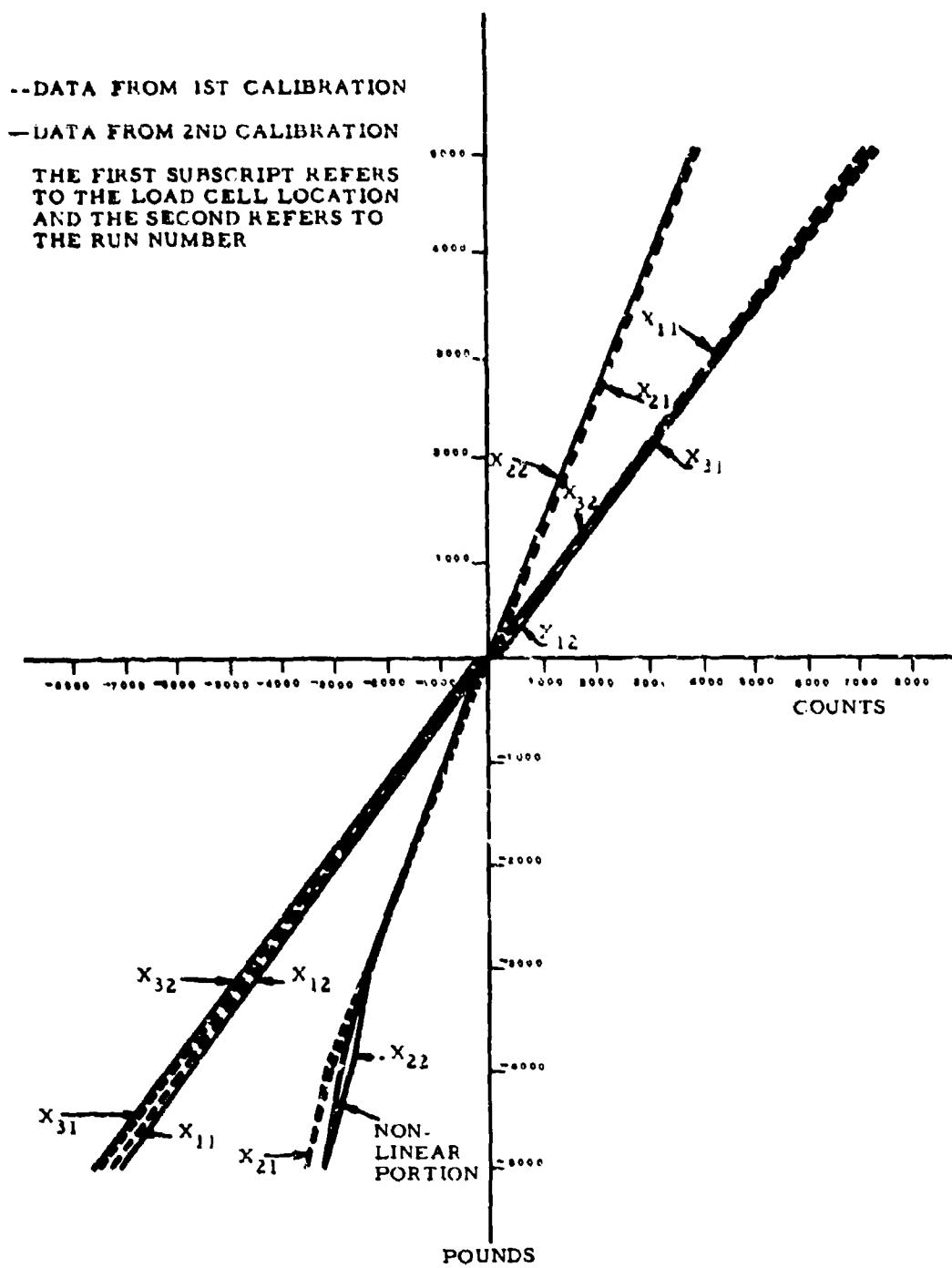


Figure 6. Comparison of First and Second Calibration, X Load Cells

## SECTION III

### ERROR ANALYSIS

#### A. ANALYSIS

The following errors were considered in this analysis: (1) bias error; (2) errors in each axis; (3) errors in the side force calculation. Figure 7 shows some parameters that were used to describe the forces produced by a TVC system, but they are not discussed in regard to error. The data for the error analysis was obtained from the accuracy verification.

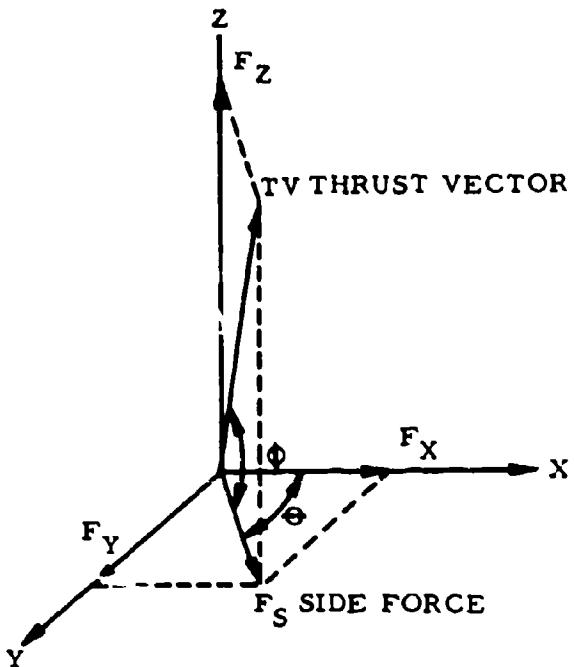


Figure 7. Vector Description of Nozzle Forces

Each error includes additively several preceding errors. For example, the error in the side force contained the errors from the X and Y axis.

### 1. Bias Error.

The data from the verification of the Gilmore thrust stand showed that all values of force derived from the calibration and data reduction process were larger than applied loads. This indicated that a bias error could be present. This bias error can be reduced by the use of a correction factor. The correction factor was obtained by the use of the following formula:

$$\text{Specific correction factor SCF} = \frac{\text{input}}{\text{average output}} = \frac{X_i}{\mu_c}$$

where input = specific load put in during the verification test =  $X_i$

$$\text{Average output} = \frac{\sum \text{outputs for each individual input}}{6} = \mu_c$$

There were 6 different outputs obtained from each input. The inputs for the X and Y axes were: 1000 lbs, 2000 lbs, 3000 lbs, 4000 lbs, 5000 lbs. The inputs for the Z axes were: 8000 lbs, 16,000 lbs, 24,000 lbs, 32,000 lbs, 40,000 lbs.

The total correction factor (TCF) was obtained by averaging the specific correction factors for each axes. The total correction factors for the three axes are:

$$\text{X axes TCF} = 0.9955$$

$$\text{Y axes TCF} = 0.9876$$

$$\text{Z axes TCF} = 1.0063$$

More specific data and calculations can be seen in the Appendix, Table III.

## 2. Errors in Each Axis.

The error in each axis is obtained first by determining the standard deviation  $\sigma$

$$\sigma = \sqrt{\frac{X_o - \mu_o}{n - 1}}$$

where  $X_o$  is the different outputs obtained from the specific input and  $\mu_o$  is the average of all the outputs as defined in Part A (1) of the error analysis. ( $n$  is 6:  $n-1$  is used because of the small sample size of 6).

The error is expressed as a  $3\sigma$  error. The error is obtained by dividing  $3\sigma$  by the average outputs for each axis.

$$\% \text{ point error} = \frac{3\sigma}{X_o} = \frac{dX}{X}$$

The % point errors are:

$$X \text{ axis error} = 0.44$$

$$Y \text{ axis error} = 0.518\%$$

$$Z \text{ axis error} = 0.18\%$$

More specific data can be seen in Table III through V in the Appendix.

## 3. Errors in the Magnitude of the Calculated Side Force.

The side force is given by the relationship  $F_s = \sqrt{F_x^2 + F_y^2}$  and the error in the side force ( $dF_s$ ) is given by  $dF_s = d(F_x^2 + F_y^2)^{1/2}$ . Let

$$F_x = X; F_y = Y; dF_s = (2Xdx + 2Ydy) \frac{1}{(X^2 + Y^2)^{1/2}}. \frac{1}{2} dF_s = \frac{Xdx + Ydy}{(X^2 + Y^2)^{1/2}}$$

where

$X = X$  side force

$Y = Y$  side force

$dx = 3\sigma$  error for  $X$  side force

$dy = 3\sigma$  error for  $Y$  side force

The % error

$$\frac{dF_s}{F_s} = \frac{XdX + YdY}{(X^2 + Y^2)^{1/2}} \cdot \frac{1}{(X^2 + Y^2)^{1/2}} = \frac{Xdx + Ydy}{X^2 + Y^2}$$

The error in the side force can be made up of many different combinations of  $X$  and  $Y$  loads and errors. This data is summarized in Table VI in the appendix. The side force error ranged from 1.14% for a side force of 1414 lbs and .33% for 7100 lbs. In general, as the side force went up, the error went down.

#### B. DISCUSSION OF ERRORS

The error analysis for the Gilmore thrust stand must not be regarded as the ultimate capability of the Gilmore stand. This error analysis was based on only one calibration and verification before the firing and one calibration after the firing. The calibration after the firing was performed to determine if the stand would give reproducible results. The stand did give reproducible results as mentioned earlier except for two cases. Because of the two cases which did not reproduce and also of the small sample size of two, the confidence limits were low. The test is very expensive (based on man-hours), and repeated testing to increase the confidence limits cannot be justified. Continued calibrations and analysis during the routine use of the thrust stand will help establish repeatability of the calibrations and confidence limits for the estimated errors of the thrust stand.

### C. ERRORS NEGLECTED

Several sources of errors were neglected. These include angular misalignment (cross axis effects) and dynamic errors.

#### 1. Angular Misalignment.

There are several types of angular misalignment that are discussed below.

a. Motor and spider are misaligned in the thrust stand while vertical load is parallel to motor centerline (Figure 8).

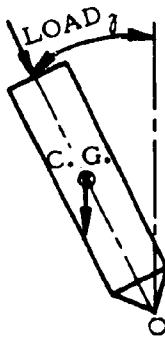


Figure 8. Motor and Spider Misalignment

In this case the offset c.g. creates a moment about O. The misaligned motor creates erroneous readings in all the load cells.

b. Motor is misaligned in thrust stand with Z spider aligned correctly while vertical load is parallel to centerline (Figure 9).

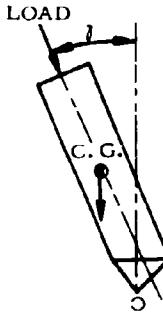


Figure 9. Motor Misalignment

In this case the load does not act through 0, thus creating a moment. Also the c.g. creates a moment about 0. All the load cell readings are affected.

c. Motor and spider are misaligned in thrust stand and load is misaligned with motor (Figure 10).

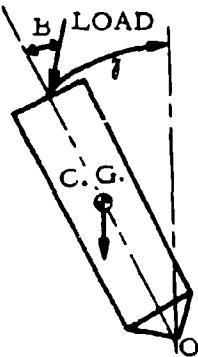


Figure 10. Motor, Spider and Load Misalignment

In this case both the load and c.g. create a moment about 0. All the load cell readings are affected.

d. Only the load is misaligned while the motor is assumed to be aligned (Figure 11).

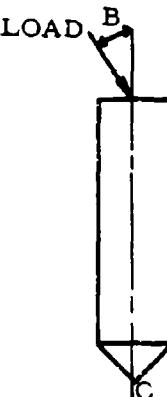


Figure 11. Load Misalignment

The misaligned load would create errors in all load cell readings. The error in the Z direction equals  $(1 - \cos \beta)$  and the error in the X and Y direction equals  $\pm \sin \beta$  in  $\beta$  (Figure 12). Only the last type of misalignment is discussed, because in the other three cases it was difficult to determine the amount of misalignment or because there were combinations of two types of misalignment.

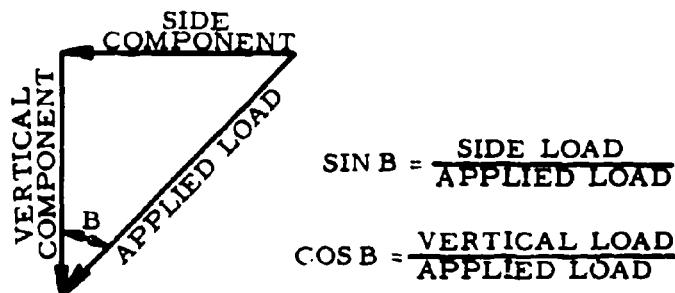


Figure 12. Side Load

A known vertical load applied to the motor at an 8-degree misalignment would cause an error in the Z axis of 1%. In contrast, an angular misalignment of only 35 minutes of arc in the vertical load would cause the X and Y load cells to read out 1% of the vertical load. This is clarified by Table II.

TABLE II. CROSS-AXIS EFFECTS

$\theta$	$100 \text{ Sin} \beta = \%$	$\text{Cos } \beta$	$100 (1 - \cos \beta) = \%$
1 min	0.00029	1.00000	0.00000
10 min	0.00290	1.00000	0.00000
35 min	0.01018	0.99995	0.00005
1 degree 9 min	0.02007	0.99980	0.00020
2 degrees 52 min	0.05001	0.99876	0.00124
8 degrees 6 min	0.14090	0.99002	0.00998

A cross-axis effect is the reading of a load cell which is not exactly perpendicular to the applied load action line. Even with a small angular misalignment, some cross-axis effects exist.

## 2. Dynamic Errors.

The response (in terms of deflection) of a linear spring-mass damper vibrational system, when excited by a time-varying force, exhibits both time and magnitude "errors" when the dynamic deflection is compared to the static deflection under the same load. In the case of a rocket motor vertical thrust stand, additional complexity results from the varying mass of the motor as propellant is consumed. Since load cell output is a direct function of load cell deflection, "errors" in deflection result directly in errors in the load cell output. While it is theoretically possible to compute the input (applied load) from the dynamic stand response (load cell reading) the multiple degrees of freedom present, as well as the nonlinear spring constants of the vibrational elements, make this approach a practical impossibility.

The practical alternative is to avoid exciting the system at frequencies near its natural frequencies, particularly the lowest natural frequency. In terms of test procedures, this requires that axial and side thrust be varied at fairly low cyclic rates (on the order of 2 to 5 cycles per second). The allowable cyclic rates could be increased by "stiffening up" the stand (i.e. by eliminating flexures and using higher-capacity load cells), but this in turn would result in poorer static-load measurement accuracy.

## SECTION IV

### RESULTS AND CONCLUSIONS

The error analysis performed on the Gilmore 50,000-lb thrust stand calibration data established that there was a .58%, 1.24%, and -.625% of point bias in the X, Y, and Z axes respectively. This was partially accounted for by the use of a correction factor. The side force magnitude error ranged from about 1.1% to .3% of point throughout the measurement range. This did not take into account cross-axis effects introduced by a single vertical load.

The major problem was adequately defining the errors in measured forces due to thrust stand interactions (cross-axis effects), changing motor weight and axial center of gravity location as a function of time. The computer program that is described in the appendix will calculate the decrease in motor weight with time, thus reducing the error associated with this factor. The error in the side force was small, less than 1.1% of point, considering the inaccuracy of the stand alignment. If the angular misalignment of the stand was known, an error could be calculated to take misalignment into account. Since it was not known, the only errors not taken into account were dynamic and misalignment errors.

## SECTION V

### RECOMMENDATIONS

The Gilmore six-component thrust stand gave acceptable results. Since dynamic errors were not taken into account, the error associated with this source was unknown. Also the amount of angular misalignment was unknown. Thus, if a more accurate estimate of the error is needed, a dynamic error analysis and an accurate stand alignment (possibly optical) should be performed. Also if this error is to be accepted with high confidence, at least four more calibrations and verifications should be performed on the test stand.

**APPENDIX**

**CALCULATIONS**

## APPENDIX

### CALCULATIONS

#### A. ERROR ANALYSIS

The following errors were considered: (1) errors in each axis; (2) error in the side force.

##### 1. Error in Each Axis.

The data from the calibration of the Gilmore thrust stand showed that all values of force applied were larger than the applied loads. Thus, there was a bias error. This bias error can be reduced by using a correction factor.

Sample Calculation:

OUTPUT (lbs)				
X INPUT	RUN 1	RUN 2	RUN 3	MODE
1000	1016.79	1005.75	1003.43	Increasing Load
2000				
5000				
1000	1005.60	1010.19	1003.08	Decreasing Load

The average for all the 1000-lb inputs was 1007.47.

Correction factor c.f. =  $1000.00/1007.47 = .9925$

In the same way correction factors were obtained for other inputs. These are shown in Tables III through V.

The correction factor for the X axis is: 0.9955

The correction factor for the Y axis is: 0.9876

The correction factor for the Z axis is: 1.0063

These were obtained by averaging the correction factors for each axis.

The  $3\sigma$  error is obtained by first averaging values for each input  $\bar{X}$ ,  $\bar{Y}$ ,  $\bar{Z}$ . From this,  $\sigma$  is obtained by the formula  $\sigma = \sqrt{\frac{\sum(X_i - \bar{X})^2}{N-1}}$  where N is the number of indicated values. N-1 is used because of the small sample size of 6.

#### Sample Calculations:

$$\bar{x} = 1007.47$$

$$x_1 - \bar{x} = 1005.60 - 1007.47 = -1.87 \text{ lbs}$$

$$x_2 - \bar{x} = 1005.75 - 1007.47 = -1.72 \text{ lbs}$$

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$$x_6 - \bar{x} = 1003.08 - 1007.47 = -4.39 \text{ lbs}$$
$$= (1.87)^2 + (1.72)^2 + \dots + (4.39)^2$$

$$\sigma_{1000} = 3.515\% \text{ point error} = 3\sigma/X_i = 10.55/1000 = 1.05\%$$

$$\sigma_{4000} = 4.51\% \text{ point error} = 3\sigma/X_i = 13.53/4000 = .34\%$$

The rest of the values are shown in Tables III through V.

TABLE III. LOAD CELL EVALUATION FOR X

LOAD (Ave of Runs)	1	2	3	4	5	6	$\sigma$	3 <sub>r</sub>	% Error	% Bias
0	-5.83	-3.87	-2.45	-4.51	-0.97	-3.58	1.68	5.036		
1000.00	-1016.79	-1005.60	-1005.75	-1010.19	-1003.43	-1003.08	3.315	10.55	1.05	0.75
(-1007.47)	-	-	-	-	-	-	-	-	-	-
2000.00	-2025.44	-2013.94	-2015.44	-2015.37	-2016.09	-2010.51	4.97	14.91	0.75	0.81
(-2016.13)	-	-	-	-	-	-	-	-	-	-
3000.00	-3025.69	-3018.41	-3017.06	-3026.44	-3018.83	-3018.02	4.16	12.48	0.42	0.69
(-3020.74)	-	-	-	-	-	-	-	-	-	-
4000.00	-4014.44	-4010.45	-4010.21	-4019.09	-4011.26	-4005.71	4.51	13.53	0.34	0.29
(-4011.86)	-	-	-	-	-	-	-	-	-	-
5000.00	-4989.81	-4979.52	-4979.43	-	-	-	-5.967	-17.91	-0.36	-0.34
(-4982.92)	-	-	-	-	-	-	Ave	0.44	0.45	

Correction factor  
= 100-.45 = .9955

Accuracy of Primary Standard = 99.99%  
Accuracy of Secondary Standard = 99.95%

TABLE IV. LOAD CELL EVALUATION FOR Y

LOAD (Ave of Runs)	1	2	3	4	5	6	$\sigma$	$3\sigma$	% Error	% Bias
0	-17.56	-16.96	-18.75	-18.95	-18.95	-19.17	1.02	3.06		
1000.0	-1008.49	-1004.90	-1010.11	-1004.74	-1015.97	-1008.95	4.10	12.30	1.23	0.89
(-1008.86)	-	-	-	-	-	-				
2000.0	-2018.19	-2014.78	-2011.26	-2013.28	-2018.31	-2013.05	2.87	8.61	0.43	0.74
(-2014.81)	-	-	-	-	-	-				
3000.0	-3040.41	-3037.27	-3037.36	-3032.37	-3040.0	-3028.85	4.51	13.53	0.45	1.20
(-3036.04)	-	-	-	-	-	-				
4000.0	-4059.17	-4055.55	-4058.95	-4057.75	-4062.54	-4058.17	2.28	6.84	0.17	1.47
(-4058.68)	-	-	-	-	-	-				
5000.0	-5094.59	-5091.70	-5101.60	-	-	-	5.09	15.27	0.31	1.91
(-5095.96)							Ave	0.52	1.24	

Correction factor  
= 100-.52 = .9948

TABLE V. LOAD CELL EVALUATION FOR Z

LOAD (Ave of Runs)	1	2	3	4	5	6	$\sigma$	$3\sigma$	% Error	% Bias
0.00	-4.31	-22.32	-24.21	-24.21	-30.18	2.795	8.39			
8000.00	Ornit 8032.84	7890.68	7897.32	7893.52	7886.89	7894.47	3.95	11.85	0.15	-1.34
(7892.58)	-	-	-	-	-	-	-	-	-	-
16000.00	15867.43	15886.54	15874.47	15847.60	15856.15	15868.49	13.65	41.01	0.26	-0.83
(15866.78)	-	-	-	-	-	-	-	-	-	-
24000.00	23863.97	23910.72	23874.47	23840.01	23842.03	23863.97	25.81	77.23	0.32	-0.56
(2385.86)	-	-	-	-	-	-	-	-	-	-
32000.00	31882.79	31914.47	31898.15	31875.11	31899.11	31898.15	13.87	41.26	0.13	-0.33
(31894.63)	-	-	-	-	-	-	-	-	-	-
40000.00	39967.90	39973.71	39980.48	-	-	-	6.31	18.91	<u>0.05</u>	<u>-0.07</u>
(39974.03)	-	-	-	-	-	-	Ave	-	-	-63

Correction factor  
 $= 100 - (-.03) = 1.0063$

Accuracy of Primary Standard = 99.99%

Accuracy of Secondary Standard = 99.95%

2. Error in the Side Force.

The side force  $F_s = \sqrt{F_x^2 + F_y^2}$

The error in the side force  $= dF_s = d(F_x^2 + F_y^2)^{1/2}$       Let  $F_x = X$   
 $F_y = Y$

$$dF_s = (2Xdx + 2Ydy) \frac{1}{2} \frac{1}{(X^2 + Y^2)^{1/2}}$$

$$dF_s = \frac{Xdx + Ydy}{(X^2 + Y^2)^{1/2}}$$

Sample calculation

$$dF_s \text{ for } X = 1000 \text{ lbs, } Y = 1000 \text{ lbs}$$

$$3\sigma 1000-x \approx 10.55 \text{ lbs}$$

$$3\sigma 1000-y \approx 12.30 \text{ lbs}$$

The above values are obtained from Tables III through V.

$$\frac{dF_s}{F_s} = \% \text{ error} \quad F_s = \frac{1}{(X^2 + Y^2)^{1/2}}$$

$$= \frac{Xdx + Ydy}{(X^2 + Y^2)^{1/2}} \cdot \frac{1}{(X^2 + Y^2)^{1/2}}$$

$$= \frac{Xdx + Ydy}{X^2 + Y^2}$$

For the above values,

$$\begin{aligned}\frac{dF_s}{F_s} &= \frac{1000(10.55) + 1000(12.30)}{1000^2 + 1000^2} \\ &= \frac{10550 + 12300}{2,000,000} \\ &= \frac{22850}{2 \times 10^6} = 1.14\%\end{aligned}$$

The rest of the values are shown in Table VI and Figure 13.

TABLE VI. SIDE FORCE BIAS AND ERROR

X LOAD X	Y LOAD Y	SIDE FORCE $F_s$	SIDE FORCE ERROR $dF_s / F_s \%$	SIDE FORCE BIAS %
1000	1000	1414	.14	.82
1000	2000	2236	.55	.74
1000	3000	3163	.11	.12
1000	4000	4123	.22	.14
1000	5000	5099	.33	.19
2000	1000	2236	.85	.82
2000	2000	2828	.59	.77
2000	3000	3605	.54	1.08
2000	4000	4472	.29	1.33
2000	5000	5385	.37	1.77
3000	1000	3163	.49	.71
3000	2000	3605	.42	.71
3000	3000	4243	.43	.95
3000	4000	5000	.26	1.19
3000	5000	5831	.33	1.58
4000	1000	4123	.39	.33
4000	2000	4472	.35	.39
4000	3000	5000	.38	.62
4000	4000	5657	.25	.88
4000	5000	6403	.31	1.29
5000	1000	5099	.39	.29
5000	2000	5385	.37	.19
5000	3000	5831	.38	.07
5000	4000	6403	.28	.36
5000	5000	7071	.33	.79

All of the values of X and Y Load were high except for one. Thus, all values of side force would be high. Thus the correction factor for the side force would be less than 1.0, i.e., Correction Factor = 1.0 - Bias Error. Figure 9 shows, in general, as the magnitude of the side force increases, the error in the magnitude of the side force decreases as shown by the points. The amount of bias is a function of the X and Y Loads for the same side force. Thus, a side force of 3605 lbs can have a bias ranging from 1.42% to 0.71%.

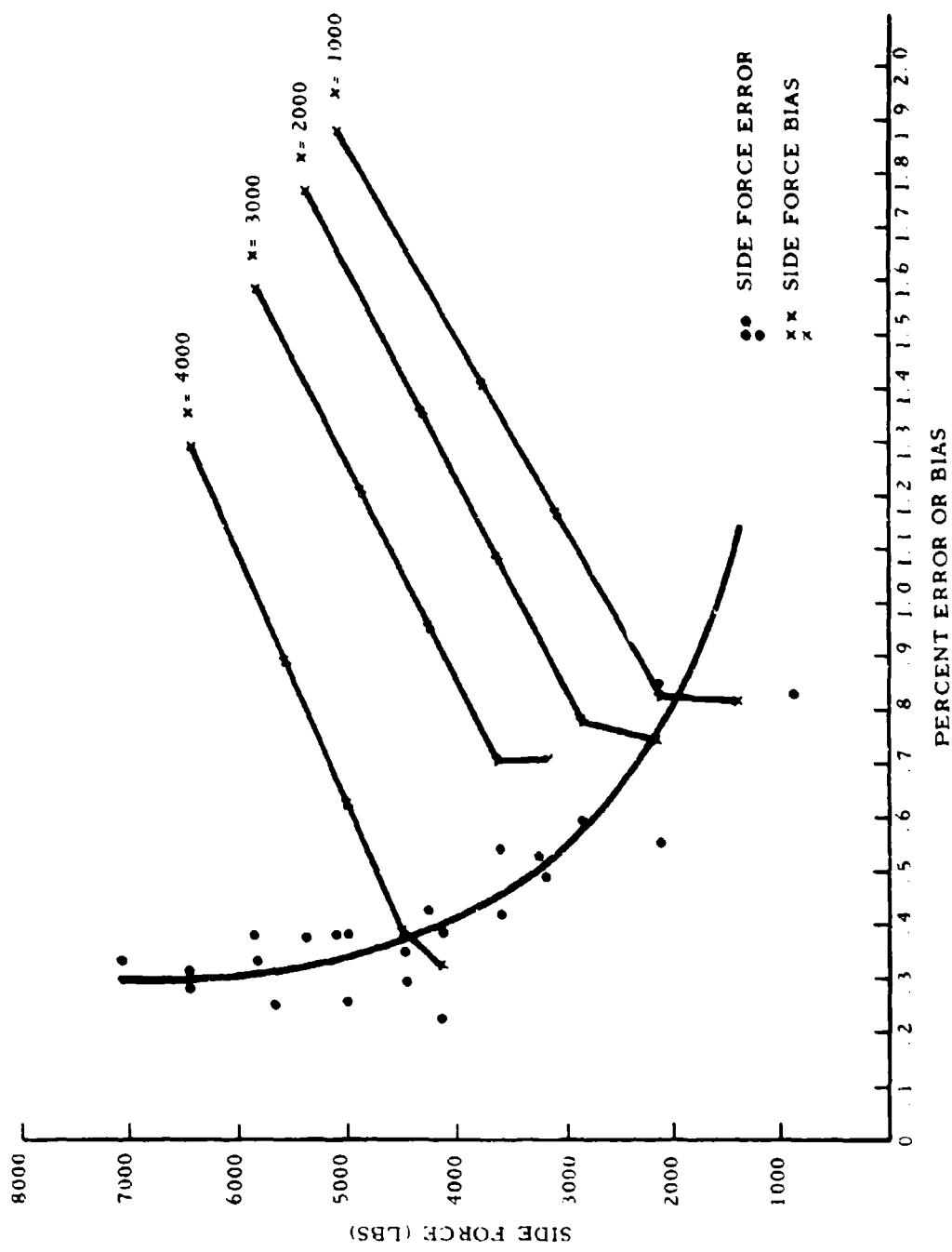


Figure 13. Side Force versus Bias Error

## B. DERIVATION OF STAND EQUATIONS

Figure 14 describes the forces on the thrust stand geometrically.

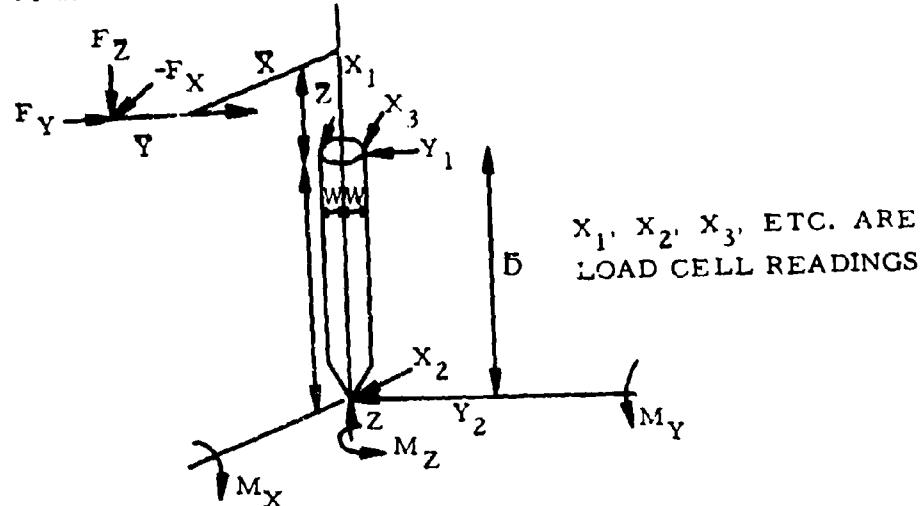


Figure 14. Stand Description

Summing forces and taking moments:

$$\sum F_x = 0 \quad F_x = X_1 + X_2 + X_3 \quad 1) \quad \sum M_x = F_y \bar{Z} - F_z \bar{Y} - Y_1 \bar{D} = 0$$

$$\sum F_y = 0 \quad F_y = Y_1 + Y_2 \quad 2) \quad \sum M_y = F_z \bar{X} - F_x \bar{Z} + (X_1 + X_3) \bar{D} = 0$$

$$\sum F_z = 0 \quad F_z = Z \quad 3) \quad \sum M_z = F_y \bar{X} - F_x \bar{Y} + (X_1 + Y_3) W = 0$$

From 1)

$$\bar{Z} = \frac{F_y \bar{Y} + Y_1 D}{F_y}$$

Sub 1) into 2

$$\frac{-F_x F_z \bar{Y} - F_x Y_1 \bar{D}}{F_y} + F_z \bar{X} + (X_1 + X_3) \bar{D} = 0$$

or 4)

$$F_z \bar{X} = \frac{F_x F_z \bar{Y} + F_x \bar{Y}_1 \bar{D}}{F_y} - (X_1 + X_3) \bar{D} = 0$$

or 4)

$$\bar{X} = \frac{F_x F_z \bar{Y} + F_x Y_1 \bar{D} - (X_1 + X_3) \bar{D} F_y}{F_y F_z} = 0$$

4) into 3

$$\bar{X} = \frac{F_x F_z \bar{Y} + F_x Y_1 \bar{D} - (X_1 + X_3) \bar{D} F_y - F_x F_z \bar{Y} + (X_1 - X_3) W}{F_z} = 0$$

$$\bar{X} = \frac{F_x Y_1 \bar{D} - (X_1 + X_3) \bar{D} F_y + (Y_1 - Y_3) W}{F_z} = 0$$

The two terms with unknowns will cancel, thus there is no solution for Y.

Try the same approach but with different values

From 2)

$$F_z \bar{X} = F_x \bar{Z} - (X_1 + X_3) \bar{D} \Rightarrow \bar{X} = \frac{F_x \bar{Z} - (X_1 + X_3) \bar{D}}{F_z}$$

into 3

$$\frac{F_y F_x \bar{Z} - F_y (X_1 + X_3) \bar{D}}{F_z} - F_x \bar{Y} + (X_1 - X_3) \bar{W} = 0$$

or

$$\bar{Y} = \frac{F_y F_x \bar{Z} - F_y (X_1 + X_3) \bar{D} + (X_1 - X_3) \bar{W} F_z}{F_x} = 0$$

into 1)

$$\frac{F_x F_y \bar{Z} - F_y F_x Z - F_y (X_1 + X_3) \bar{D} + (X_1 - X_3) \bar{W} F_z}{F_x} = 0$$

Again the two terms with unknowns cancel each other. Thus, the values for two of the components cannot be computed. The only one that can be determined is Z.

Thus, taking out the components X and Y, Z can be computed.

$$\Sigma M_x = Y_1 \bar{D} - F_y \bar{Z} = 0$$

$$\Sigma M_y = (X_1 + X_3) \bar{D} - F_x \bar{Z} = 0$$

$$\Sigma M_z = (X_1 + X_3) \bar{W} = 0$$

Solving for Z

$$Z = \frac{Y_1 \bar{D}}{Y_1 + Y_2} \quad \text{or} \quad Z = \frac{(X_1 + X_3) \bar{D}}{(X_1 + X_2 + X_3)}$$

These stand equations give the location of point of application of the thrust vector.

### C. COMPUTER PROGRAM

A computer program was written to provide the following items:

1. Magnitude and direction of side force vector.
2. Axial thrust corrected to given mass flow rate.
3. Ratio of resolved side force to axial thrust.
4. Amplification factor.

During a firing, propellant is being consumed, and the center of gravity is constantly changing. The weight of the expended propellant exhausted was determined by a computer program. The correct value of thrust was obtained in the following manner:

Weight of propellant exhausted at anytime  $t = W_{pt}$  (lbs) = burn rate  
lbs/sec  $\times$  incremented time  $T$  (sec) = (lbs)

Force at anytime read by load cells:  $t = F_t$

Total weight of propellant at start =  $F_p$

Correct Z force =  $F_z - (F_p - W_{pt})$  = corrected thrust at any time  $t$ .

The thrust values obtained from the load cells are corrected and the true thrust is plotted on graph paper as thrust versus time.

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13. ABSTRACT This report describes the calibration and an error analysis of the 6-component 50,000-pound Gilmore Rocket Motor Thrust Stand located on Pad 2, Air Force Rocket Propulsion Laboratory, Solid Test Area (I-32). The purpose of this report is to establish stand accuracy and reproducibility for a calibrated static test loading.		
Errors considered primarily resulted from thrust stand misalignment, propellant mass flow of the motor, and cross-axis interactions. Dynamic effects resulting from vibration were not included in the detailed analysis but are briefly discussed.		

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